CHAPTER 11 Study Guide and Review

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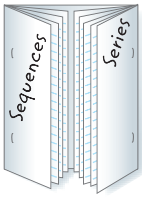
Be sure the following Key Concepts are noted in your Foldable.



Key Concepts

Arithmetic Sequences and Series (Lessons 11-1 and 11-2)

- The nth term a_n of an arithmetic sequence with first term a_1 and common difference d is given by $a_n = a_1 + (n - 1)d$, where n is any positive integer.
- The sum S_n of the first n terms of an arithmetic series is given by $S_n = \frac{n}{2}[2a_1 + (n - 1)d]$ or $S_n = \frac{n}{2}(a_1 + a_n)$.

Geometric Sequences and Series (Lessons 11-3 to 11-5)

- The nth term a_n of a geometric sequence with first term a_1 and common ratio r is given by $a_n = a_1 \cdot r^{n-1}$, where n is any positive integer.
- The sum S_n of the first n terms of a geometric series is given by $S_n = \frac{a_1(1 - r^n)}{1 - r}$ or $S_n = \frac{a_1 - a_1 r^n}{1 - r}$, where $r \neq 1$.
- The sum S of an infinite geometric series with $-1 < r < 1$ is given by $S = \frac{a_1}{1 - r}$.

Recursion and Special Sequences (Lesson 11-6)

- In a recursive formula, each term is formulated from one or more previous terms.

The Binomial Theorem (Lesson 11-7)

- The Binomial Theorem:

$$(a + b)^n = \sum_{k=0}^{n} \frac{n!}{(n-k)!k!} a^{n-k} b^k$$

Mathematical Induction (Lesson 11-8)

- Mathematical induction is a method of proof used to prove statements about the positive integers.

Key Vocabulary

arithmetic means (p. 624)
arithmetic sequence (p. 622)
arithmetic series (p. 629)
Binomial Theorem (p. 665)
common difference (p. 622)
common ratio (p. 636)
convergent series (p. 651)
factorial (p. 666)
Fibonacci sequence (p. 658)
geometric means (p. 638)
geometric sequence (p. 636)
geometric series (p. 643)
index of summation (p. 631)
inductive hypothesis (p. 670)
infinite geometric series (p. 650)
iteration (p. 660)
mathematical induction (p. 670)
partial sum (p. 650)
Pascal's triangle (p. 664)
recursive formula (p. 658)
sequence (p. 622)
series (p. 629)
sigma notation (p. 631)
term (p. 622)

Vocabulary Check

Choose the term from the list above that best completes each statement.

1. A(n) __________ of an infinite series is the sum of a certain number of terms.

2. If a sequence has a common ratio, then it is a(n) __________ .

3. Using __________ , the series $2 + 5 + 8 + 11 + 14$ can be written as $\sum_{n=1}^{5} (3n - 1)$.

4. Eleven and 17 are two __________ between 5 and 23 in the sequence 5, 11, 17, 23.

5. Using the __________ , $(a - 2)^4$ can be expanded to $a^4 - 8a^3 + 24a^2 - 32a + 16$.

6. The __________ of the sequence 3, 2, $\frac{4}{3}$, $\frac{8}{9}$, $\frac{16}{27}$ is $\frac{2}{3}$.

7. The __________ $11 + 16.5 + 22 + 27.5 + 33$ has a sum of 110.

8. A(n) __________ is expressed as $n! = n(n - 1)(n - 2) \ldots 2 \cdot 1$.

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Lesson-by-Lesson Review

11–1 Arithmetic Sequences (pp. 622-628)

Find the indicated term of each arithmetic sequence.

9. $a_1 = 6, d = 8, n = 5$
10. $a_1 = -5, d = 7, n = 22$
11. $a_1 = 5, d = -2, n = 9$
12. $a_1 = -2, d = -3, n = 15$

Find the arithmetic means in each sequence.

13. -7, ___, ___, ___, 9
14. 12, ___, ___, 4
15. 9, ___, ___, ___, ___, -6
16. 56, ___, ___, ___, 28

17. **GLACIERS** The fastest glacier is recorded to have moved 12 kilometers every three months. If the glacier moved at a constant speed, how many kilometers did it move in one year?

Example 1 **Find the 12th term of an arithmetic sequence if $a_1 = -17$ and $d = 4$.**

$a_n = a_1 + (n - 1)d$ Formula for the nth term

$a_{12} = -17 + (12 - 1)4$ $n = 12, a_1 = -17, d = 4$

$a_{12} = 27$ Simplify.

Example 2 **Find the two arithmetic means between 4 and 25.**

$a_n = a_1 + (n - 1)d$ Formula for the nth term

$a_4 = 4 + (4 - 1)d$ $n = 4, a_1 = 4$

$25 = 4 + 3d$ $a_4 = 25$

$7 = d$ Simplify.

The arithmetic means are 4 + 7 or 11 and 11 + 7 or 18.

11–2 Arithmetic Series (pp. 629-635)

Find S_n for each arithmetic series.

18. $a_1 = 12, a_n = 117, n = 36$
19. $4 + 10 + 16 + \ldots + 106$
20. $10 + 4 + (-2) + \ldots + (-50)$
21. Evaluate $\sum_{n=2}^{13} (3n + 1)$.

22. **PATTERNS** On the first night of a celebration, a candle is lit and then blown out. The second night, a new candle and the candle from the previous night are lit and blown out. This pattern of lighting a new candle and all the candles from the previous nights is continued for seven nights. Find the total number of candle lightings.

Example 3 **Find S_n for the arithmetic series with $a_1 = 34$, $a_n = 2$, and $n = 9$.**

$S_n = \frac{n}{2}(a_1 + a_n)$ Sum formula

$S_9 = \frac{9}{2}(34 + 2)$ $n = 9, a_1 = 34, a_n = 2$

$= 162$ Simplify.

Example 4 **Evaluate $\sum_{n=5}^{11} (2n - 3)$.**

Use the formula $S_n = \frac{n}{2}(a_1 + a_n)$. There are 7 terms, $a_1 = 2(5) - 3$ or 7, and $a_7 = 2(11) - 3$ or 19.

$S_7 = \frac{7}{2}(19 + 7)$

$= 91$

11-3 Geometric Sequences (pp. 636-641)

Find the indicated term of each geometric sequence.

23. $a_1 = 2, r = 2, n = 5$

24. $a_1 = 7, r = 2, n = 4$

25. $a_1 = 243, r = -\frac{1}{3}, n = 5$

26. a_6 for $\frac{2}{3}, \frac{4}{3}, \frac{8}{3}, \ldots$

Find the geometric means in each sequence.

27. 3, ____ , ____ , 24

28. 7.5, ____ , ____ , ____ , 120

29. SAVINGS Kathy has a savings account with a current balance of \$5000. What would Kathy's account balance be after five years if she receives 3% interest annually?

Example 5 Find the fifth term of a geometric sequence for which $a_1 = 7$ and $r = 3$.

$a_n = a_1 \cdot r^{n-1}$ Formula for the nth term

$a_5 = 7 \cdot 3^{5-1}$ $n = 5, a_1 = 7, r = 3.$

$a_5 = 567$ The fifth term is 567.

Example 6 Find two geometric means between 1 and 8.

$a_n = a_1 \cdot r^{n-1}$ Formula for the nth term

$a_4 = 1 \cdot r^{4-1}$ $n = 4$ and $a_1 = 1$

$8 = r^3$ $a_4 = 8$

$2 = r$ Simplify.

The geometric means are 1(2) or 2 and 2(2) or 4.

11-4 Geometric Series (pp. 643-649)

Find S_n for each geometric series.

30. $a_1 = 12, r = 3, n = 5$

31. $4 - 2 + 1 - \ldots$ to 6 terms

32. $256 + 192 + 144 + \ldots$ to 7 terms

33. Evaluate $\sum_{n=1}^{5} \left(-\frac{1}{2}\right)^{n-1}$.

34. TELEPHONES Joe started a phone tree to give information about a party to his friends. Joe starts by calling 3 people. Then each of those 3 people calls 3 people, and each person who receives a call then calls 3 more people. How many people have been called after 4 layers of the phone tree? (*Hint:* Joe is considered the first layer.)

Example 7 Find the sum of a geometric series for which $a_1 = 7$, $r = 3$, and $n = 14$.

$S_n = \frac{a_1 - a_1 r^n}{1 - r}$ Sum formula

$S_{14} = \frac{7 - 7 \cdot 3^{14}}{1 - 3}$ $n = 14, a_1 = 7, r = 3$

$S_{14} = 16{,}740{,}388$ Use a calculator.

Example 8 Evaluate $\sum_{n=1}^{5} \left(\frac{3}{4}\right)^{n-1}$.

$S_5 = \frac{1\left[1 - \left(\frac{3}{4}\right)^5\right]}{1 - \frac{3}{4}}$ $n = 5, a_1 = 1, r = \frac{3}{4}$

$= \frac{\frac{781}{1024}}{\frac{1}{4}}$ $\frac{3}{4}^5 = \frac{243}{1024}$

$= \frac{781}{256}$

Mixed Problem Solving
For mixed problem-solving practice, see page 936.

11-5 Infinite Geometric Series (pp. 650-655)

Find the sum of each infinite geometric series, if it exists.

35. $a_1 = 6, r = \frac{11}{12}$

36. $\frac{1}{8} - \frac{3}{16} + \frac{9}{32} - \frac{27}{64} + \cdots$

37. $\sum_{n=1}^{\infty} -2\left(-\frac{5}{8}\right)^{n-1}$

38. GEOMETRY If the midpoints of the sides of $\triangle ABC$ are connected, a smaller triangle results. Suppose the process of connecting midpoints of sides and drawing new triangles is continued indefinitely. Find the sum of the perimeters of all of the triangles if the perimeter of $\triangle ABC$ is 30 centimeters.

Example 9 **Find the sum of the infinite geometric series for which $a_1 = 18$ and $r = -\frac{2}{7}$.**

$S = \frac{a_1}{1 - r}$ — Sum formula

$= \frac{18}{1 - \left(-\frac{2}{7}\right)}$ — $a_1 = 18, r = -\frac{2}{7}$

$= \frac{18}{\frac{9}{7}}$ or 14 — Simplify.

11-6 Recursion and Special Sequences (pp. 658-662)

Find the first five terms of each sequence.

39. $a_1 = -2, a_{n+1} = a_n + 5$

40. $a_1 = 3, a_{n+1} = 4a_n - 10$

Find the first three iterates of each function for the given initial value.

41. $f(x) = -2x + 3, x_0 = 1$

42. $f(x) = 7x - 4, x_0 = 2$

43. SAVINGS Toni has a savings account with a \$15,000 balance. She has a 4% interest rate that is compounded monthly. Every month Toni makes a \$1000 withdrawal from the account to cover her expenses. The recursive formula $b_n = 1.04b_{n-1} - 1000$ describes the balance in Toni's savings account after n months. Find the balance of Toni's account after the first four months. Round your answer to the nearest dollar.

Example 10 **Find the first five terms of the sequence in which $a_1 = 2, a_{n+1} = 2a_n - 1$.**

$a_{n+1} = 2a_n - 1$	Recursive formula
$a_{1+1} = 2a_1 - 1$	$n = 1$
$a_2 = 2(2) - 1$ or 3	$a_1 = 2$
$a_{2+1} = 2a_2 - 1$	$n = 2$
$a_3 = 2(3) - 1$ or 5	$a_2 = 3$
$a_{3+1} = 2a_3 - 1$	$n = 3$
$a_4 = 2(5) - 1$ or 9	$a_3 = 5$
$a_{4+1} = 2a_4 - 1$	$n = 4$
$a_5 = 2(9) - 1$ or 17	$a_4 = 9$

The first five terms of the sequence are 2, 3, 5, 9, and 17.

Study Guide and Review

11-7 The Binomial Theorem (pp. 664-669)

Expand each power.

44. $(x - 2)^4$

45. $(3r + s)^5$

Find each indicated term of each expansion.

46. fourth term of $(x + 2y)^6$

47. second term of $(4x - 5)^{10}$

48. **SCHOOL** Mr. Brown is giving a four-question multiple-choice quiz. Each question can be answered A, B, C, or D. How many ways could a student answer the questions using each answer A, B, C, or D once?

Example 11 Expand $(a - 2b)^4$.

$(a - 2b)^4$

$$= \sum_{k=0}^{4} \frac{4!}{(4-k)!k!} a^{4-k}(-2b)^k$$

$$= \frac{4!}{4!0!} a^4(-2b)^0 + \frac{4!}{3!1!} a^3(-2b)^1 + \frac{4!}{2!2!} a^2(-2b)^2 + \frac{4!}{1!3!} a^1(-2b)^3 + \frac{4!}{0!4!} a^0(-2b)^4$$

$$= a^4 - 8a^3b + 24a^2b^2 - 32ab^3 + 16b^4$$

11-8 Proof and Mathematical Induction (pp. 670-674)

Prove that each statement is true for all positive integers.

49. $1 + 2 + 4 + \ldots + 2^{n-1} = 2^n - 1$

50. $6^n - 1$ is divisible by 5.

51. $3^n - 1$ is divisible by 2.

52. $1 + 4 + 7 + \ldots (3n - 2) = \frac{n(3n - 1)}{2}$

Find a counterexample for each statement.

53. $n^2 - n + 13$ is prime.

54. $13^n + 11$ is divisible by 24.

55. $9^{n+1} - 1$ is divisible by 16.

56. $n^2 + n + 1$ is prime.

Example 12 Prove that $1 + 5 + 25 + \cdots + 5^{n-1} = \frac{1}{4}(5^n - 1)$ for positive integers n.

Step 1 When $n = 1$, the left side of the given equation is 1. The right side is $\frac{1}{4}(5^1 - 1)$ or 1. Thus, the equation is true for $n = 1$.

Step 2 Assume that $1 + 5 + 25 + \cdots + 5^{k-1} = \frac{1}{4}(5^k - 1)$ for some positive integer k.

Step 3 Show that the given equation is true for $n = k + 1$.

$1 + 5 + 25 + \ldots + 5^{k-1} + 5^{(k+1)-1}$

$= \frac{1}{4}(5^k - 1) + 5^{(k+1)-1}$ Add to each side.

$= \frac{1}{4}(5^k - 1) + 5^k$ Simplify the exponent.

$= \frac{5^k - 1 + 4 \cdot 5^k}{4}$ Common denominator

$= \frac{5 \cdot 5^k - 1}{4}$ Distributive Property

$= \frac{1}{4}(5^{k+1} - 1)$ $5^k = 5^{k+1}$

Thus, the equation is true for $n = k + 1$. The conjecture is proved.